

The light σ -meson

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In the framework of the dispersion relation N/D -approach, we restore the low-energy $\pi\pi$ ($IJ^{PC} = 00^{++}$)-wave amplitude sewing it with the previously obtained K -matrix solution for the region 450–1900 MeV. The restored N/D -amplitude has a pole on the second sheet of the complex- s plane near the $\pi\pi$ threshold, that is the light σ -meson.

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At present the understanding of scalar meson is one of the key problems for the Strong-QCD physics. The $\pi\pi$ low-mass data provide indications on the existence of a low-mass σ -meson. This state is beyond $q\bar{q}$ and gluonium systematics, which makes it necessary to confirm its existence as well as to study the possible mechanisms of its formation.

Experimental data on meson spectra accumulated by the Crystal Barrel Collaboration [1], GAMS [2] and BNL [3] groups provided a good basis for setting up the $q\bar{q}$ /gluonium classification of the light scalars. For the ($IJ^{PC} = 00^{++}$)-wave, the combined K -matrix analysis of the reactions $\pi\pi \rightarrow \pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\pi\pi\pi\pi$ has been carried out over the mass range 450–1900 MeV [4,5], then the K -matrix analysis was extended to the waves $\frac{1}{2}0^+$ [6] and 10^+ [5], thus making it possible to establish the $q\bar{q}$ systematics of scalars for $1^3P_0q\bar{q}$ and $2^3P_0q\bar{q}$ multiplets.

The advantage of the K -matrix representation is that it allows us not only to determine the locations and partial widths of resonances but also to study characteristics of corresponding states with switched off decay channels: these "primary states", or "bare states" (i.e. states without a cloud of mesons produced by decay processes) are suitable objects for the $q\bar{q}$ /gluonium classification (see [7] for the details). The decay processes in the scalar/isoscalar sector cause a strong mixing which destroys the $q\bar{q}$ /gluonium classification. Another important effect generated by transitions $(q\bar{q})_1 \rightarrow \text{real mesons} \rightarrow (q\bar{q})_2$ is an accumulation of widths of neighbouring resonances by one of them, that results in appearance of a broad state.

According to [4,5], five bare scalar/isoscalar states are located in the region 700–1800 MeV: $f_0^{\text{bare}}(720 \pm 100)$, $f_0^{\text{bare}}(1230 \pm 50)$, $f_0^{\text{bare}}(1260 \pm 30)$, $f_0^{\text{bare}}(1600 \pm 50)$ and $f_0^{\text{bare}}(1800 \pm 30)$. Four of them are members of the $q\bar{q}$ nonets $1^3P_0q\bar{q}$ and $2^3P_0q\bar{q}$ while one state, $f_0^{\text{bare}}(1260 \pm 30)$ or $f_0^{\text{bare}}(1600 \pm 50)$, is the lightest glueball; results of the lattice calculations [8] tell us that it is $f_0^{\text{bare}}(1600 \pm 50)$. After the mixing originated from the decay processes, the primary, or bare states are transformed into a set of resonances: $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1530^{+90}_{-250})$ and $f_0(1750)$. The state $f_0(1530^{+90}_{-250})$ is rather broad, its large width is due to the accumulation of widths of neighbouring resonances: the gluonium and $q\bar{q}$ states are strongly mixed because the transition $\text{gluonium} \rightarrow q\bar{q}$ is not suppressed in terms of the rules of $1/N$ expansion [9]. The gluonium component is shared between the the broad state $f_0(1530^{+90}_{-250})$ and the scalars $f_0(1370)$ and $f_0(1500)$.

An important result of the article [4,5] is that the K -matrix 00^{++} -amplitude has no pole singularities in the region 500–800 MeV. Here the $\pi\pi$ -scattering phase δ_0^0 increases smoothly reaching 90° at 800–900 MeV. A straightforward explanation of such a behaviour of δ_0^0 might consist in the presence of a broad resonance, with a mass about 600–900 MeV and width $\Gamma \sim 500$ MeV (for example, see Refs. [10,11] and references therein). However, according to the K -matrix solution [4,5], the 00^{++} -amplitude does not contain pole singularities on the second sheet of the complex- $M_{\pi\pi}$ plane inside the interval $450 \leq \text{Re } M_{\pi\pi} \leq 900$ MeV: the K -matrix amplitude has a low-mass pole only, which is located on the second sheet either near the $\pi\pi$ threshold or even below it. In [4,5], the presence of this pole was not emphasized, for the left-hand cut, which is important for the reconstruction of analytic structure of the low-energy partial amplitude, was taken into account only indirectly; a proper way for the description of the low-mass amplitude must be the dispersion relation representation.

In this paper the $\pi\pi$ -scattering N/D -amplitude is reconstructed in the region of small $M_{\pi\pi}$ being sewed with the K -matrix solution [4,5] found for $M_{\pi\pi} \sim 450 - 1950$ MeV. More specifically, using the data for δ_0^0 , we construct the N/D amplitude below 900 MeV sewing it with the K -matrix amplitude, bearing in mind to make a continuation of the amplitude into the region $s = M_{\pi\pi}^2 \sim 0$. With this sewing we strictly follow the results obtained for the K -matrix amplitude in the region 450–900 MeV, that is, the region where we can trust the K -matrix representation of the amplitude. Recall that the K -matrix representation allows us to restore correctly the analytic structure of the amplitude in the region $s > 0$ (by taking into account threshold and pole singularities) but not the left-hand singularities at $s \leq 0$ (where singularities are related to forces). Therefore, being cautious, we cannot be quite confident of the K -matrix results below $\pi\pi$ threshold.

The dispersion relation amplitude is reconstructed with the method of approximation of the left-hand cut suggested

in [12]. The found N/D -amplitude provides us with a good description of δ_0^0 from the threshold to 900 MeV, that includes the region where $\delta_0^0 \sim 90^\circ$. This amplitude does not have a pole in the region 500–900 MeV; instead, the pole is located near the $\pi\pi$ threshold. This pole corresponds to the light σ -meson.

I. DISPERSION RELATION REPRESENTATION FOR THE $\pi\pi$ SCATTERING AMPLITUDE BELOW 900 MEV

The partial pion-pion scattering amplitude being a function of the invariant energy squared $s = M_{\pi\pi}^2$ can be represented as a ratio $N(s)/D(s)$ where $N(s)$ has a left-hand cut which is due to "forces" (interactions due to t - and u -channel exchanges), while $D(s)$ is determined by the rescatterings in the s -channel. $D(s)$ is given by the dispersion integral along the right-hand cut in the complex- s plane:

$$A(s) = \frac{N(s)}{D(s)}, \quad D(s) = 1 - \int_{4\mu_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\rho(s')N(s')}{s' - s - i0}. \quad (1)$$

Here $\rho(s)$ is the invariant $\pi\pi$ phase space, $\rho(s) = (16\pi)^{-1} \sqrt{(s - 4\mu_\pi^2)/s}$. It was supposed in (1) that $D(s) \rightarrow 1$ with $s \rightarrow \infty$ and CDD-poles are absent (a detailed presentation of the N/D -method can be found in [13]).

The N -function can be written as an integral along the left-hand cut as follows:

$$N(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{L(s')}{s' - s}, \quad (2)$$

where the value s_L marks the beginning of the left-hand cut. For example, for the one-meson exchange contribution $g^2/(m^2 - t)$, the left-hand cut starts at $s_L = 4\mu_\pi^2 - m^2$, and at this point the N -function has a logarithmic singularity; for the two-pion exchange $s_L = 0$.

Below we deal with the amplitude $a(s)$ which is defined as follows:

$$a(s) = \frac{N(s)}{8\pi\sqrt{s} \left(1 - P \int_{4\mu_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\rho(s')N(s')}{s' - s} \right)}. \quad (3)$$

The amplitude $a(s)$ is related to the scattering phase shift: $a(s)\sqrt{s/4 - \mu_\pi^2} = \tan \delta_0^0$. In Eq. (3) the threshold singularity is singled out explicitly, so the function $a(s)$ contains only the left-hand cut together with poles corresponding to zeros of the denominator of the right-hand side in (3) which follow from: $1 = P \int_{4\mu_\pi^2}^{\infty} (ds'/\pi) \cdot \rho(s')N(s')/(s' - s)$.

The pole of $a(s)$ at $s > 4\mu_\pi^2$ corresponds to the phase shift value $\delta_0^0 = 90^\circ$. The phase of the $\pi\pi$ scattering reaches the value $\delta_0^0 = 90^\circ$ at $\sqrt{s} = M_{90} \simeq 850$ MeV. Because of that, the amplitude $a(s)$ may be represented in the form:

$$a(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\alpha(s')}{s' - s} + \frac{C}{s - M_{90}^2} + D. \quad (4)$$

To reconstruct the low-mass amplitude the parameters D, C, M_{90} and $\alpha(s)$ have been determined by fitting to the experimental data. In the fit we have used a method, which has been established in the analysis of the low-energy nucleon-nucleon amplitudes [12]. Namely, the integral in the right-hand side of (4) has been replaced by the sum

$$\int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\alpha(s')}{s' - s} \rightarrow \sum_n \frac{\alpha_n}{s_n - s} \quad (5)$$

with $-\infty < s_n \leq s_L$.

In the fit to the data for δ_0^0 at $\sqrt{s} \leq 950$ MeV, see Fig. 1a, we impose the following constraints on the N/D -solution in order to sew it to the K -matrix amplitude found previously at $\sqrt{s} \sim 450$ –1950 MeV [4,5]:

(i) The N/D -solution curve for $a(s)$, see Fig. 1b, should be inside the corridor determined by the K -matrix solution at $450 \text{ MeV} \leq \sqrt{s} \leq 950 \text{ MeV}$: the corridor in Fig. 1b is shown by the error bars for the K -matrix solution points.

(ii) The N/D -amplitude should be analytical (not having pole singularities) in the following complex- s region on the second sheet: $0.25 \text{ GeV}^2 \leq \text{Re } s \leq 0.8 \text{ GeV}^2$ and $0 \leq \text{Im } s \leq 0.6 \text{ GeV}^2$.

The description of data within the N/D -solution, which uses six terms in the sum (5), is demonstrated on Fig. 1a. The parameters of the solution are as follows (all values are in μ_π units):

$$M_{90} = 6.228, \quad C = -13.64, \quad D = 0.316, \quad (6)$$

$$(\alpha_n, s_n) = (2.23, -9.56), (2.21, -10.16), (2.19, -10.76),$$

$$(0.247, -32), (0.246, -36), (0.245, -40).$$

The scattering length found for this solution is equal to $a_0^0 = 0.22 \mu_\pi^{-1}$ (experiment gives $a_0^0 = 0.26 \pm 0.05 \mu_\pi^{-1}$ [14]), the Adler zero is at $s = 0.12 \mu_\pi^2$. The N/D -amplitude is sewed with the K -matrix amplitude of Refs. [4,5], and figure 1b demonstrates the level of coincidence for the amplitudes $a(s)$ for both solutions (the values of $a(s)$ which correspond to the K -matrix amplitude are shown with error bars determined in [4,5]).

The dispersion relation solution has correct analytic structure at the region $|s| < 1 \text{ GeV}^2$. The amplitude has no poles on the first sheet of the complex- s plane; the left-hand cut of the N -function after the replacement given by Eq. (5) is transformed into a set of poles on the negative piece of the real s -axis: six poles of the amplitude (at $s/\mu_\pi^2 = -5.2, -9.6, -10.4, -31.6, -36.0, -40.0$) represent the left-hand singularity of $N(s)$. On the second sheet (under the $\pi\pi$ -cut) the amplitude has two poles: at $s \simeq (4 - i14)\mu_\pi^2$ and $s \simeq (70 - i34)\mu_\pi^2$ (see Fig. 2). The second pole, at $s = (70 - i34)\mu_\pi^2$, is located beyond the region under consideration for which $|s| < 1 \text{ GeV}^2$ (nevertheless, let us stress that the K -matrix amplitude [4,5] has a set of poles just in the region of the second pole of the N/D -amplitude). The pole near the threshold, at

$$s \simeq (4 - i14)\mu_\pi^2, \quad (7)$$

is what we discuss. The N/D -amplitude has no poles at $\text{Re}\sqrt{s} \sim 600 - 900 \text{ MeV}$ despite the phase shift δ_0^0 reaches 90° here.

In the solution discussed above, Eq. (6), the left-hand singularity is described by six poles. With this number of poles, the solution is weakly depending on their change: for example, the five-pole solution with $a_0^0 = 0.22 \mu_\pi^{-1}$ gives practically the same result at $\text{Re } s > 0$ as the six-pole one.

The data do not fix the N/D -amplitude rigidly. The position of the low-mass pole can be easily varied in the region $\text{Re } s \sim (0 - 4)\mu_\pi^2$, and there are subsequent variations of the scattering length in the interval $a_0^0 \sim (0.21 - 0.28)\mu_\pi^{-1}$ and Adler zero at $s \sim (0 - 1)\mu_\pi^2$. Ambiguities in fixing the σ -pole and Adler zero positions are mainly due to comparatively large error bars in the measured δ_0^0 near threshold, at $\sqrt{s} < 350 \text{ MeV}$.

Let us stress that the way of reconstruction of the dispersion relation amplitude used here differs from the mainstream attempts to determine the N/D -amplitude. In the classical N/D procedure, that is the bootstrap one, the pion-pion amplitude is to be determined by analyticity, unitarity and crossing symmetry. This means a unique determination of the left-hand cut by the crossed channels. However the bootstrap procedure is not carried out till now; the problems which faces the nowadays bootstrap program are discussed in Ref. [15] and references therein. Nevertheless, one can try to saturate the left-hand cut by known resonances in the crossing channels. Usually one supposes that the dominant contribution to the left-hand cut comes from the ρ -meson exchange supplemented by $f_2(1275)$ and σ exchanges. Within this scheme the low-energy amplitude is restored, being controlled by the available experimental data.

In the scheme used here the amplitude in the physical region at 450-950 MeV is supposed to be known from the K -matrix analysis, and then a continuation of the amplitude is performed from 450-900 MeV to the region of smaller masses; the continuation is restricted by the data. As a result, we restore the pole near the threshold (the low-mass σ -meson) and the left-hand cut (although with less accuracy, actually on a qualitative level).

In the approaches which take into account the left-hand cut as a contribution of certain meson exchanges, the following locations of the low-mass pole were obtained:

- (i) dispersion relation approach, $s \simeq (0.2 - i22.5)\mu_\pi^2$ [16],
- (ii) meson exchange models, $s \simeq (3.0 - i17.8)\mu_\pi^2$ [17], $s \simeq (0.5 - i13.2)\mu_\pi^2$ [18], $s \simeq (2.9 - i11.8)\mu_\pi^2$ [19],
- (iii) linear σ -model, $s \simeq (2.0 - i15.5)\mu_\pi^2$ [20].

However, in [21–26], the pole positions were found in the region of higher s , at $s > 7\mu_\pi^2$, that reflects ambiguities of the approaches which treat the left-hand cut as a known quantity.

II. CONCLUSION

On the basis of the dispersion relation N/D -representation, we have continued the K -matrix 00^{++} amplitude found previously for $\sqrt{s} = M_{\pi\pi} \sim 450 - 1950$ MeV [4,5] to the $\pi\pi$ threshold region, $s \sim (0 - 4\mu_\pi^2)$; the continuation procedure has been corrected by the low-energy data. The amplitude found in this way has a pole near the $\pi\pi$ threshold, at $Re\ s \sim (0 - 4\mu_\pi^2)$; this pole corresponds to the light σ -meson. This result is in a qualitative agreement with that of Refs. [16–20] where the analysis of the 00^{++} amplitude was performed by modelling the left-hand cut contribution.

With the results for the K -matrix analysis of Refs. [4,5], one has six scalar/isoscalar states in the region below 1800 MeV. Five of them are descendants of the $q\bar{q}$ states ($1^3P_0 q\bar{q}$ and $2^3P_0 q\bar{q}$) and gluonium. They are $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1530^{+30}_{-250})$ and $f_0(1750)$. Three states, $f_0(1370)$, $f_0(1500)$ and $f_0(1530^{+30}_{-250})$, shared the gluonium component. There are arguments (see Ref. [7] for details) that the broad state $f_0(1530^{+30}_{-250})$ is a descendant of the lightest scalar gluonium which mass, according to lattice calculations [8], is in the region 1500–1700 MeV; an appearance of the broad state $f_0(1530^{+30}_{-250})$ is due to the specific effect of accumulation of widths of the overlapping resonances [27]. So, we conclude that the light σ -meson is beyond both the $q\bar{q}$ and gluonium systematics.

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- [1] V.V. Anisovich et al., Phys. Lett. **B323** 233 (1994);
C. Amsler et al., Phys. Lett. **B333** 277 (1994); **B355** 425 (1995).
 - [2] D. Alde et al., Phys. Lett. **B241** 600 (1990); Zeit. Phys. **C66** 375 (1995);
Yu.D. Prokoshkin et al., Physics-Doklady **342** 473 (1995);
F. Binon et al., Nuovo Cim. **A78** 313 (1983); **A80** 363 (1984).
 - [3] S.J. Lindenbaum and R.S. Longacre, Phys. Lett. **B274** 492 (1992);
A. Etkin et al., Phys. Rev. **D25** 1786 (1982).
 - [4] V.V. Anisovich, Yu.D. Prokoshkin and A.V. Sarantsev, Phys. Lett. **B389** 388 (1996).
 - [5] V.V. Anisovich, A.A. Kondashov, Yu.D. Prokoshkin, S.A. Sadovsky, and A.V. Sarantsev, *The two-pion spectra for the reaction $\pi^- p \rightarrow \pi^0 \pi^0 n$ at 38 GeV/c pion momentum and combined analysis of the GAMS, Crystal Barrel and BNL data*, hep-ph/9711319, Phys. of Atomic Nuclei (in press).
 - [6] A.V. Anisovich and A.V. Sarantsev, Phys. Lett. **B413** 137 (1997).
 - [7] A.V. Anisovich, V.V. Anisovich and A.V. Sarantsev, Zeit. Phys. **A359** 173 (1997).
 - [8] G.S. Bali et al., Phys. Lett. **B309** 378 (1993);
J. Sexton, A. Vaccarino and D. Weingarten, Phys. Rev. Lett. **75** 4563 (1995);
C.J. Morningstar and M. Peardon, Phys. Rev. **D56** 4043 (1997).
 - [9] G. 't'Hooft, Nucl. Phys. **B72** 161 (1974);
G. Veneziano, Nucl. Phys. **B117** 519 (1976).
 - [10] L. Montanet, *Scalars*, Talk at Euroconference *QCD-99*, Montpellier, France, 1999.
 - [11] P. Minkowski and W. Ochs *Identification of the glueballs and the scalar meson nonet of lowest mass*, Preprint MPI-PhT/98-89, hep-ph/9811518.
 - [12] V.V. Anisovich, M.N. Kobrinsky, D.I. Melikhov and A.V. Sarantsev, Nucl. Phys. **A544** 747 (1992).
 - [13] G.F. Chew, *The analytic S-matrix*, NY, Benjamin, 1966.
 - [14] L. Rosselet et al., Phys. Rev. **D15** 576 (1977);
O. Dumbraits et al., Nucl. Phys. **B216** 277 (1983).
 - [15] A.V. Vereshagin and V.V. Vereshagin, Phys. Rev. **D58**:016002 (1999).
 - [16] J.L. Basdevant, C.D. Frogatt and J.L. Petersen, Phys. Lett. **B41** 178 (1972).
 - [17] J.L. Basdevant and J. Zinn-Justin, Phys. Rev. **D3** 1865 (1971);
D. Iagolnitzer, J. Justin and J.B. Zuber, Nucl. Phys. **B60** 233 (1973).
 - [18] B.S. Zou and D.V. Bugg, Phys. Rev. **D48** R3942 (1994); **D50** 591 (1994).
 - [19] G. Janssen, B.C. Pearce, K. Holinde and J. Speth, Phys. Rev. **D52** 2690 (1995).
 - [20] N.N. Achasov and G.N. Shestakov, Phys. Rev. **D49** 5779 (1994).
 - [21] S.D. Protopopescu et al. Phys. Rev. **D7** 1279 (1973).
 - [22] P. Estabrooks, Phys. Rev. **D19** 2678 (1979)
 - [23] K.L. Au, D. Morgan and M.R. Pennington, Phys. Rev. **D35** 1633 (1987).
 - [24] S. Ishida et al., Prog. Theor. Phys. **98** 1005 (1997).

- [25] N.A. Törnquist and M. Roos, Phys. Rev. Lett. **76** 1575 (1996).
- [26] M.P. Locher, V.E. Markushin and H.Q. Zheng, Eur. Phys. J. **C4** 317 (1998).
- [27] V.V. Anisovich, D.V. Bugg and A.V. Sarantsev, Phys. Rev. **D58**:111503 (1998).

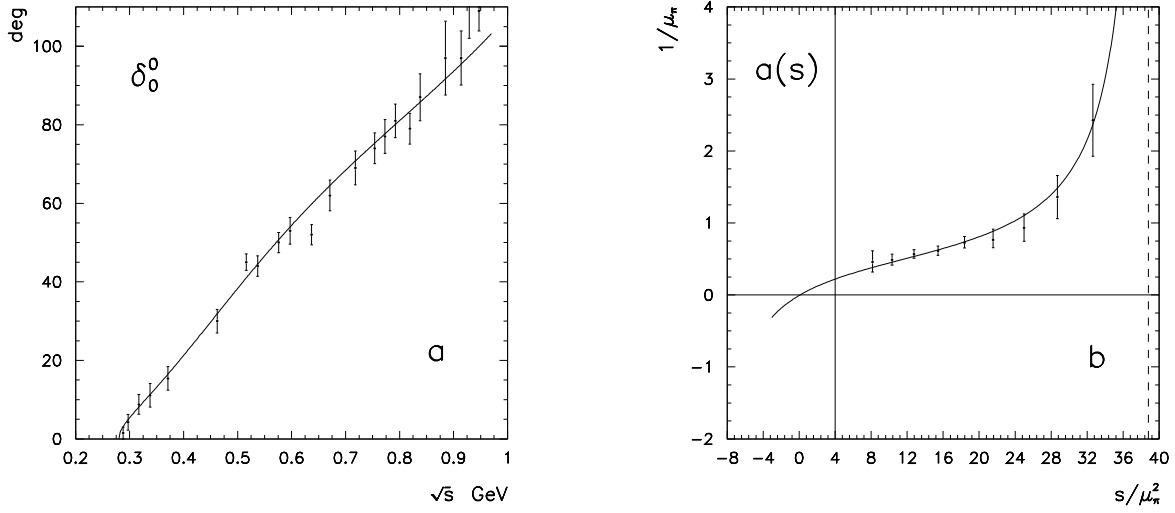


FIG. 1. a) Fit to the data on δ_0^0 by using the N/D -amplitude. b) Amplitude $a(s)$ in the N/D -solution (solid curve) and the K -matrix approach [4,6] (points with error bars).

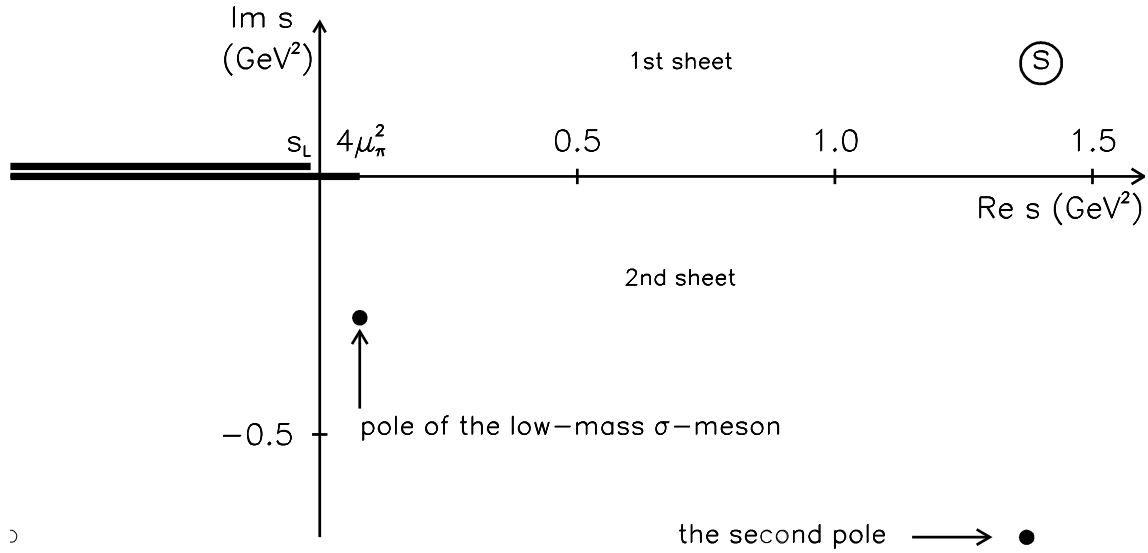


FIG. 2. Complex-s plane and singularities of the N/D -amplitude